Heat flow in a 2D slab: A MATLAB based estimation

Ambreen Aslam1*, Shaukat Ali Shahid1*, Ahsan Raza Sattar2
Sobia Zareen1, Lubna Younas1, Ahmed Azam1

1Department of Physics, University of Agriculture Faisalabad-38040, Pakistan
2Department of Computer Sciences, University of Agriculture Faisalabad-38040, Pakistan

Abstract
This paper reports the estimation of a basic phenomenon of nature heat flow by solving a set of equations. The flow taken here is two dimensional and the third dimension is intentionally neglected for a more simple calculation. To analyze the heat transfer from its surroundings, a 2D rectangular slab of any material, is divided into a grid (of any desired size i.e. nxn), and then to find the heat flow through grid, Laplace's equation for heat flow is employed. The calculations involve numerous differential equations and their huge iterations for solutions, it's not possible to figure out the results manually for large grid sizes, a MATLAB code for the problem is developed to estimate and visualize the temperature variations and transfer of heat at different points of the grid on the slab, whereas the effect of grid size on estimation of heat flow is also discussed.

Key words: Heat flow; 2D slab; MATLAB

1. Introduction

Heat transfer processes are integral to buildings and all energy systems including automobiles, heat exchangers, electronic products, fuel cells and power plants [1-3]. Besides, the performance of the building structures exposed to intensely changing outdoor climate: sunlight, wind rain, and ambient temperature requires energy to realize the desired indoor climate. Previously, the building plans were based upon the experience. However, due to the inflexible desires on performance, and also the considerable increase of new building skills, new materials and new building designs, this dependence on experience is not always applicable anymore and may result in building damage, a ruthless indoor climate and excessive energy consumption. Thus, the understanding of heat transport through building structures is much essential for building design [4-7].

A very little has been published in heat transfer research area. Therefore, it is imperative to make contribution in this potential area of research to cope with global energy challenges. A MATLAB code for the problem is developed to estimate and visualize the temperature variations and transfer of heat at different points of the grid on the slab along with the effect of grid size on estimation of heat flow in order to find the stabilized temperature at the different unknown points of the 2D grid.

2. Material and Methods

To analyze and estimate the heat flow through a 2D slab, an iterative method is used, which can be found by the following equation [8,9].

\[
U(i,j)=1/4\{(U(i+1,j)+U(i-1,j)+U(i,j+1)+U(i,j-1)), \tag{1}
\]

This is Laplace’s Equation for Heat Flow, where \(U\) represents the temperature. This gives a grid in \(x\ y\) plane where each grid is identified by a set of indices \((i,\ j)\). Then applied this method into MATLAB and got results for many iterations, which gives more precise results, i.e. more closer to the analytically solved results \([10,11]\).

2.1. ALGORITHM

This is the algorithm followed to develop a code in MATLAB.

Start

STEP 1>Take initial values of temperatures from the user for all four sides of the 2D object.

STEP 2>Ask user to select any number of cells i.e. \(n\) to which the grid will be divided.

STEP 3>Assign the temperatures to their corresponding grid points.
STEP 4> Assign initially zero value to all the internal unknown points of the grid.
STEP 4> Iterate the formula by given n and their indices using the user’s inputs.
STEP 5> Assign the results obtained to their corresponding points each time after iteration.
STEP 6> Check by a condition whether the iterated result matches its previous value or not?
STEP 7> If condition gives YES then show these values as the final results. And if NO then keep doing the same as mentioned in STEP 4-6 until it says YES to condition.
End

3. Results and Discussion

The purpose of this MATLAB code is to find the stabilized temperature at the different unknown points of the 2D grid. When the user of this code enters the required inputs, there begins a process of iterations in the system, which will be continued for any number of times for which the given conditions remains true, and immediately after the conditions get false, the control transferred to the next upcoming statements of the code, which are the output of the code. For example Figure 1.(a) shows a grid that is made up of 20x20 square boxes, each box have unique points and their names, all these point names are assigned to their separate equations. Figure 1(b) shows the variations in temperature contour at different points of grid, Figure 1(c) is representing the flow of heat point to point. The inputs by the user is as follows:

--------------------------USER INPUTS--------------------------

Enter THE DIMENSION OF GRID i.e. “n”; 20
Enter the temp. for the Right side of the grid in degree Celsius (°C): 65
Enter the temp. for the Left side of the grid in degree Celsius (°C): 43
Enter the temp. for the Bottom side of the grid in degree Celsius (°C): 39
Enter the temp. for the Top side of the grid in degree Celsius (°C): 13

Fig. 1 (a): 20 x 20 grid on a slab. (b) Temperature contour on a 20 x 20 grid slab. (c) Heat flow representation in a 20 x 20 slab
Fig. 2 (a) shows an example of grid that is made up of 10x10 square boxes, each box have unique points and their names, all these point names are assigned to their separate equations. Fig. 2 (b) shows variations in the temperature contour at different points of grid; Fig. 2 (c) is representing the flow of heat point to point. The inputs by the user are as follows:

--------------------- USER INPUTS ---------------------

Enter THE DIMENSION of GRID i.e. “n”: 10

Enter the temp. for the Right side of the grid in degree Celsius (°C) : 50

Enter the temp. for the Left side of the grid in degree Celsius (°C) : 35

Enter the temp. for the Bottom side of the grid in degree Celsius (°C) : 24

Enter the temp. for the Top side of the grid in degree Celsius (°C) : 18

Conclusions

To evaluate the temperature variations and transfer of heat through MATLAB grid on the 2D slab, Laplace’s equation for heat flow is employed and a code for the problem is developed to estimate and visualize the temperature variations and transfer of heat at different points of the grid on the slab. The results directly depend upon the grid size, i.e., how many cells are used to divide the slab in a grid. If the grid size is larger the results would be more accurate and vice versa. Also the material used effects the results, as if we use a glass slab and an iron slab both considered with same boundary temperatures, same number of cells of grid, but at the end we would get different results. That’s not much surprising as there exists a different kind of atomic structure in every element or substance, so the heat flow would be different for them. And that gives different values of temperature, the conductive materials gets hotter easily, as compared to insulators.

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References


